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Development and Validation of Linear and Non-Linear Model of Passive Suspension System

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Abstract: *In this paper the validation of both linear and non-linear behaviour of the spring is studied. Mathematical model for the suspension system is created and equation of motion is derived. Passive suspension system is taken into consideration for the validation. Two approaches are performed i.e. State space and SIMULINK. The State space equation is derived from the mathematical model for the linear and non-linear suspension systems. Simulation of the model is created in MATLAB SIMULINK for both the systems. The analytical model is validated with SIMULINK model. Both the results of the State space and MATLAB SIMULINK are validated.*
Keywords *- Linear, Non-Linear, Passive suspension, Simulink, Quarter car*

I. Introduction

The suspension system plays the vital role in the vehicle. It is the indivisible part in the whole vehicle. Its main significance is to provide ease during the travelling period. The suspension minimizes the discomfort originated by the vibrating body of the vehicle. Quarter car model has the two degrees of freedom, which consists of the spring and the damper. The spring stiffness and the damping coefficient of the damper are considered.For designing of the suspension system it is necessary to consider the non-linear parameters of the spring and the damper. Harmonic motion is given as an input to the shock absorber system. The results are linear and non-linear system is compared [1]. A.G.Mohite, A.C.Mitra developed the non-linear model for the quarter car system for passive suspension .The paper mainly focused on the non-linear parameters of the system and created the state space model and the Simulink model .The model is validated with the results obtained and found that the non-linear model has more improvement as compared to the linear model. The various values of linear parameters are considered [2]. B.D.Mahajan, A. A.Divekar analysed the behaviour of the quarter car model with the help of the sine wave as the input and mathematical model is derived. It is seen that there is decrease in the overshoot and the settling time with increase in frequency of the system [3]. Two degree of freedom mathematical model is obtained with the help of Newton's second law and the passive suspension system is connected in loop and is validated using the simulation [4].A.C.Mitra, T.Soni, Kiranchand. G.R has taken into consideration the influential factor of the ride comfort. Quarter car test rig has been developed for the optimization of the suspension system [5]. K.K.Jagtap, D.R.Dolas has considered the quarter car passive suspension system for the analysis. The result of the simulation as well as the experimental results is variable due to the various non linear properties of the system

In the further section of the paper the development of the quarter car model is shown. The mathematical model is developed with the help of the equations derived from the Newton's second law of motion. Firstly it is derived for the linear model and then for the non-linear model of the suspension system. The two approaches of the Analytical (State space) and the SIMULINK approach is performed .With the help of this approaches the graph and results are obtained to validate the linear as well as the non-linear model of the suspension system.

II. Quarter Car Model

The quarter car model has the two degrees of freedom. It consists of the spring stiffness, damping coefficient and tire stiffness. The SIMULINK model is developed on the basis of the quarter car model. The Fig.1 and Fig.2 shows the Quarter car models for linear suspension system and Non-linear suspension system respectively. The symbols in the Fig.1 indicate Mass of Sprung body (Ms), Mass of Un -sprung body (Mu), Spring stiffness (Ks), Damping Coefficient (Cs), Damping Stiffness of tire (Kt) .The symbols in the Fig.2 indicates Non-linear spring stiffness (Ks1 and Ks2), Non-linear damping coefficient (Cs1 and Cs2), Non-linear tire stiffness (Kt1,Kt2 and Kt2), Xs, Xu and Xr are the sprung mass displacement, unsprung mass displacement and tire displacement respectively caused by the step input.

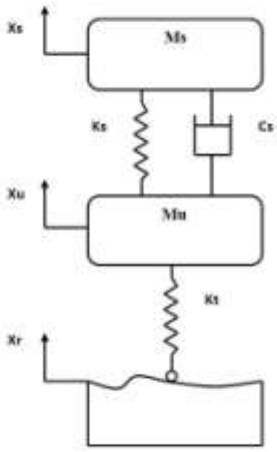

Fig.1 Linear Model

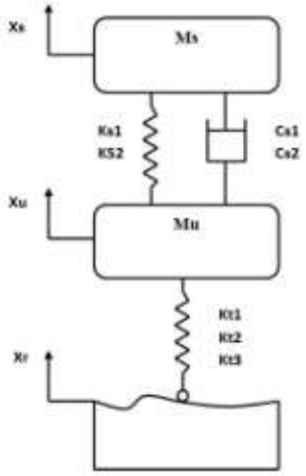

Fig.2 Non-linear Model

III. Mathematical Modeling

Linear Model

The equation (1) and equation (2) is derived from the Newton's second law of motion

$$M_S \ddot{X}_s = -K_s(X_s - X_u) - C_s(\dot{X}_s - \dot{X}_u) \quad (1)$$

$$M_u \ddot{X}_u = K_s(X_s - X_u) + C_s(\dot{X}_s - \dot{X}_u) - K_t(X_u - X_r) \quad (2)$$

The above equations (1) consist of the terms like $\ddot{X}_s$, $\dot{X}_s$ and X_s are acceleration, velocity and displacement for the sprung mass. The equation (2) consists of the terms like $\ddot{X}_u$, Xu is the acceleration and the velocity.

Table no 1-Linear Parameters of suspension

Parameters of suspension	**Symbol**	**Unit**	**Value**
Mass of sprung body	Ms	kg	295
Mass of unsprung body	Mu	kg	39
Spring Stiffness	Ks	N/m	2031
Damping Coefficient	Cs	Nm/s	9015
Tire Stiffness	Kt	N/m	41815.66

For the validation of the model the values in the Table 1 are taken into consideration to calculate the displacement of the unsprung mass.

Analytical solution (State space)

State space is nothing but a theoretical approach towards the solution. State space equation is derived from the above equations (1) and (2) obtained from the mathematical model. It is the analytical method to obtain the results for the validation.

$$\dot{X} = Ax + Bu$$

$$\begin{Bmatrix} dx1 \\ dx2 \\ dx3 \\ dx4 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -Ks/Ms & Ks/Ms & -Cs/Ms & Cs/Ms \\ Ks/Mu & -(Ks+Kt)/Mu & Cs/Mu & -Cs/Mu \end{bmatrix} \begin{Bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{Bmatrix}$$

$$Y = Cx + Du$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{Bmatrix} + [0]Zr$$

Where A defines State Matrix, B defines Input Matrix, C defines Output Matrix and D defines Direct Transmission Matrix.

Matlab Simulink Model For Linear Suspension System

The Fig.3 shows the simulation model developed in the MATLAB/SIMULINK. This simulation model is useful in getting the graph of the displacement vs. time to check the settling time of the system. Simulation is nothing but the blocks use to generate the signals of various inputs to get the similar type of result as compared to the experimental results.

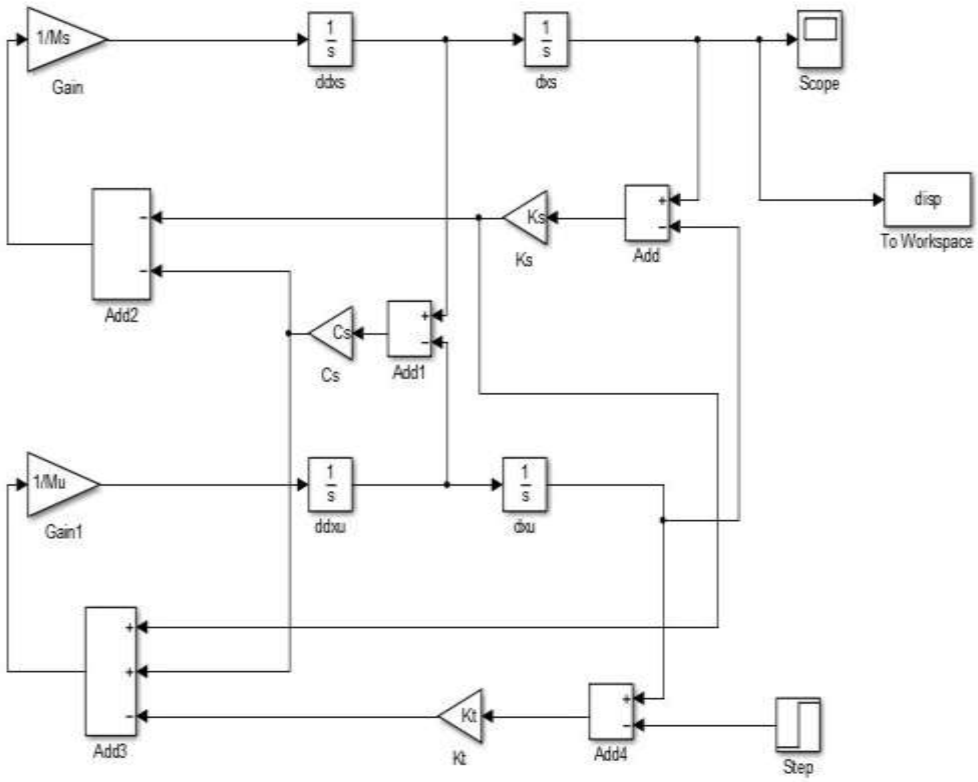

Fig.3 Simulation Model for Linear Suspension System

Non-linear Model

$$F_t = K_{t1}(X_u - X_r) + K_{t2}(X_u - X_r)^2 - K_{t3}(X_u - X_r)^3$$
$$F_s = K_{s1}(X_s - X_u) + K_{s2}(X_s - X_u)^2$$
$$F_d = C_{s1}(\dot{X}_s - \dot{X}_u) + C_{s2}(\dot{X}_s - \dot{X}_u)^2$$

Where Ft is the Tire force, Fs is the spring force and Fd is the damping force.

$$M_S \ddot{X}_s = -K_{s1}(X_s - X_u) - K_{s2}(X_s - X_u)^2 - C_{s1}(\dot{X}_s - \dot{X}_u) - C_{s2}(\dot{X}_s - \dot{X}_u)^2 \quad (3)$$
$$M_u \ddot{X}_u = K_{s1}(X_s - X_u) + K_{s2}(X_s - X_u)^2 + C_{s1}(\dot{X}_s - \dot{X}_u) + C_{s2}(\dot{X}_s - \dot{X}_u)^2 - K_{t1}(X_u - X_r) - K_{t2}(X_u - X_r)^2 + K_{t3}(X_u - X_r)^3 \quad (4)$$

The above equations (3) consist of the terms like$\ddot{X}_s$,$\dot{X}_s$ and X_s are acceleration, velocity and displacement for the sprung mass. The equation (4) consists of the terms like$\ddot{X}_u$,Xu are the acceleration and displacement.

For the validation of the non-linear model the values from the Table 2 is used.

State Space Approach

State space equation is derived from the above equations (3) and (4) obtained from the mathematical model. It is the analytical method to obtain the results for the validation.

Table no 2: Suspension Parameters for non-linear system

Parameters of suspension	Symbol	Unit	Value
Mass of sprung body	Ms	kg	295
Mass of unsprung body	Mu	kg	39
Linear Damping Coefficient	Cs1	Ns/m	3482
Square Damping for Non-linear	Cs2	Ns/m^2	580
Linear Spring Stiffness	Ks1	N/m	15302
Spring Stiffness for Non-Linear	Ks2	N/m^2	2728
Linear Tire Stiffness	Kt1	N/m	60063
Square Tire Stiffness for Non-linear	Kt2	N/m^2	42509
Cube Tire Stiffness for Non-Linear	Kt3	N/m^3	22875

$$\dot{X} = AX + Bu$$

$$\begin{Bmatrix} dx1 \\ dx2 \\ dx3 \\ dx4 \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -(Ks1+Ks2)/Ms & Ks1+Ks2/Ms & -(Cs1+Cs2)/Ms & Cs1+Cs2/Ms \\ Ks1+Ks2/Mu & -(Ks1+Ks2+Kt1)/Mu & Cs1+Cs2/Mu & -(Cs1+Cs2)/Mu \end{bmatrix} \begin{Bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{Bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ (Kt1+Kt2+Kt3)/Mu \end{bmatrix} \{Zr\}$$

$$Y = Cx + Du$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} x1 \\ x2 \\ x3 \\ x4 \end{Bmatrix} + [0]Zr$$

Where A defines State Matrix, B defines Input Matrix, C defines Output Matrix and D defines Direct Transmission Matrix.

SIMULINK Model Of Non-Linear Suspension System

The fig.4 shows the simulation model developed in the MATLAB/SIMULINK. Simulation model helps to get the result of the simulation and graph for the non-linear model is obtained. This simulation model is useful in getting the graph of the displacement vs. time to check the settling time of the system. Simulation is nothing but the blocks use to generate the signals of various inputs to get the similar type of result as compared to the experimental results.

IV. Validation

Fig.5 shows the validation of linear suspension system which contains both the solution for the State space approach and MATLAB/SIMULINK approach. It consists of the combination of the state space and the Simulink of the graph is for the step input. It is clearly visible that the nature of both the graph of SIMULINK and the state space approach is similar.

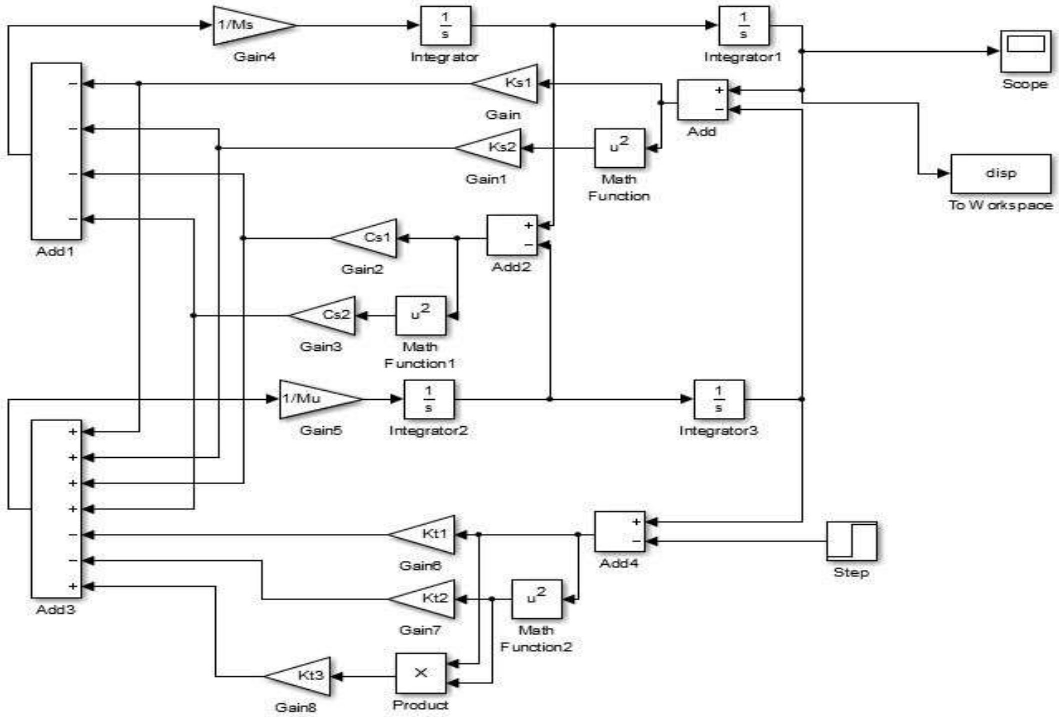

Fig.4 Simulation Model for non-linear suspension system

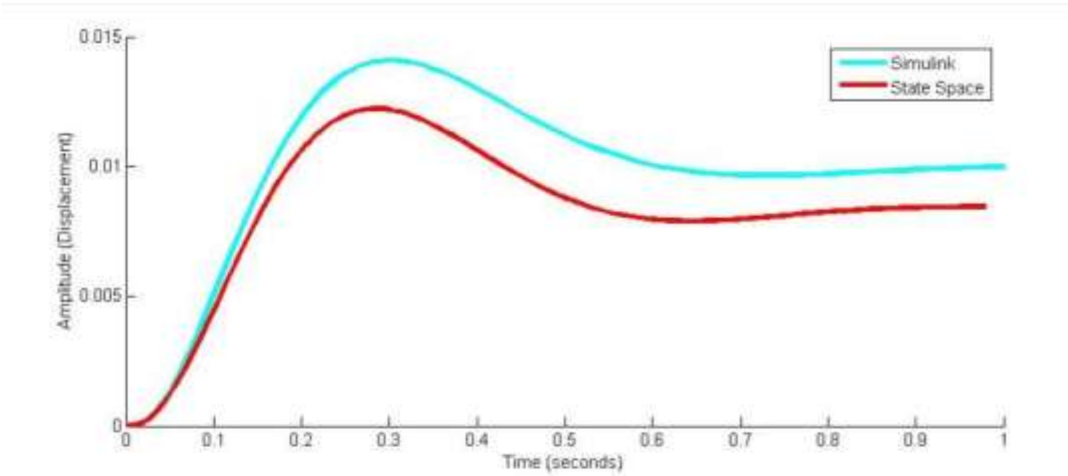

Fig.6 Validation of non-linear model

V. Results and Conclusion

The step input is given to the linear model. The Fig.5 shows the linear model, the behavior of the graph is exactly same for the State Space approach and the MATLAB/SIMULINK. Hence the validation of the linear model is done. For the non-linear model the step input is given to the system.Fig.6 shows the nature of the graph for the non-linear model. There is the slight difference in the graph because of the elimination of the higher order in the equation for the simplification of the solution. Both the linear and non-linear models are validated with the help of the State space approach and the MATLAB/SIMULINK.

Analyzing the suspension system is the main objective by considering the linear and non-linear model of the suspension system. The main purpose of the validation is to analyze and validate the linear as well as non-linear model by using the effective parameters like Ks, Cs, Ms and Mu and after this validation considering it into the design of the suspension system.

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